GEOPHYSICAL RESEARCH LETTERS, VOL. 22, NO. 16, PAGES 2301-2304, AUGUST 15, 1995

Constraining odd-degree Earth structure with coupled free-oscillations

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Abstract. Several pairs of normal mode multiplets sensitive to long-wavelength mantle structure are potential coupling partners. Geometric selection rules render such coupling the only means by which normal modes sample odd-degree mantle structure. That overtone coupling can be strong and can significantly impact normal mode spectra is illustrated using synthetic seismograms for a realistic earth model. These observations motivate the generalization of normal mode spectral fitting to permit simultaneous estimation of even- and odd-degree structural constraints. When generalized spectral fitting is applied to a data set of approximately 150 high signal-to-noise vertical records, dominated by data from the Northern Bolivian event (6/9/1994), it is found to reduce misfits appreciably for coupled multiplets ${}_1S_5-{}_2S_4$ and ${}_2S_5-{}_1S_6$, to yield odd-degree constraints generally consistent with recent mantle models, and to improve agreement between estimated and predicted even-degree constraints.

Introduction

Free oscillation studies to date have focused on the observation and analysis of multiplet center frequencies and line widths [e.g., Smith & Masters, 1989] and on the singlet frequencies and self-coupling interaction coefficients estimated by singlet stripping and spectral fitting techniques [Ritzwoller *et al.*, 1988; Giardini *et al.*, 1988; Li *et al.*, 1991; Widmer & Masters, 1992]. These studies have provided increasingly accurate constraints on even-degree Earth structures.

Normal mode sensitivity to odd-degree structure, however, is provided only through the interaction of the singlets of nearly degenerate multiplets. In addition, when coupling is significant but unmodeled, estimation of even degree constraints can be strongly biased. These considerations motivate the generalization of spectral fitting to incorporate inter-multiplet coupling.

This paper has three purposes:

- to illustrate the impact of odd-degree structures, through inter-multiplet coupling, on normal mode spectra and on the estimation of normal mode structural constraints;
- to demonstrate the existence and effectiveness of a generalized spectral fitting technique that incorporates inter-multiplet coupling;
- to present the first estimates of odd-degree structural constraints by generalized spectral fitting.

The results of this study demonstrate the necessity and efficacy of applying generalized spectral fitting to a variety of normal mode analysis problems.

Modal Coupling: Forward and Inverse Theory

Starting from a spherically symmetric Earth model, designated by $\mathbf{m}_0(r)=[\kappa_0(r),\mu_0(r),\rho_0(r)]$, an isotropic elastic Earth model is described by $\mathbf{m}(r)=\mathbf{m}_0(r)+\delta\mathbf{m}(r,\theta,\phi)$. The perturbations to the spherical model may be represented as sums of spherical harmonic components, $\delta\mathbf{m}(r)=\sum_{s,t}[\delta\kappa_s^t(r),\delta\mu_s^t(r),\delta\rho_s^t(r)]Y_s^t(\theta,\phi)$, where s and t index, respectively, the angular and azimuthal orders of the harmonics of lateral structure.

Two free oscillation multiplets of the model $\mathbf{m}_0(r)$, indexed by their radial and angular orders $k=(n,l)$ and $k'=(n',l')$, comprise $2l+1$ and $2l'+1$ degenerate singlet modes, referred to by azimuthal indices m and m'. Assuming the pair of multiplets is well isolated in frequency from other modes, the corresponding modes of the aspherical Earth model are linear combinations of the spherical earth modes, as given by the eigenvectors of the interaction matrix, $Z_{nn'll'}^{mm'}=C_{nn'l}^{m}\delta_{ll'}\delta_{mm'}+\sum_{s,t}\Gamma_{ll's}\,c_{s(nn'll')}^{t}$, with singlet frequencies determined from the matrix eigenvalues. The C term includes the impacts of multiplet spacing and of the Earth's rotation and ellipticity, while the Γ factors are algebraic functions that result from the geometry of the spherical harmonic basis functions. These factors multiply interaction coefficients of the form, $c_{s(kk')}^{t}=\int_0^a \delta\mathbf{m}_s^t(r)\cdot\mathbf{M}_{s(kk')}(r)\,r^2dr$, with structure kernels, $\mathbf{M}_s(r)=[K_s(r),M_s(r),R_s(r)]$, that are known combinations of the radial eigenfunctions [Woodhouse, 1980]. The interaction coefficients summarize the impact of structural perturbations on the modes. For self-coupling, $k=k'$, $\Gamma=0$ for even s, and the interaction coefficients constrain only even-degree structure. For inter-multiplet (cross-) coupling of spheroidal modes, $k\neq k'$ and $\Gamma=0$ for $(l+l'+s)$ odd. Thus, cross-coupling interaction coefficients for overtone pairs such as ${}_1S_5-{}_2S_4$ and ${}_2S_5-{}_1S_6$ constrain only structures of odd harmonic degree, s.

In spectral fitting, as described by Ritzwoller *et al.* [1988], Giardini *et al.* [1988], and Li *et al.* [1991], the interaction coefficients are estimated by linearizing their effect on the data. The difference between the spectrum of recording j and the nth iteration best-fit synthetic spectrum is

Paper number 95GL01996
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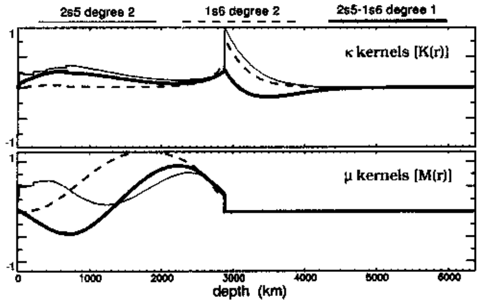

Figure 1. Degrees 1 and 2 structure kernels for ${}_2S_5{-}{}_1S_6$ [Woodhouse, 1980].

$$s_j^{\mathrm{data}}(\omega_i)-s_j^{(n)}(\omega_i)=\sum_{k,k'}\sum_{s,t}\left[\frac{\partial s_j^{(n)}(\omega_i)}{\partial c^t_{s(kk')}}\delta c^{t\,(n)}_{s(kk')}\right] \quad (1)$$

for each discrete frequency (index i) in the band around the target pair of multiplets. The vector $\delta\mathbf{c}^{(n)}$ comprises perturbations to interaction coefficients. This is equivalent to the matrix equation $\Delta\mathbf{s}^{(n)}=\mathbf{A}^{(n)}\,\delta\mathbf{c}^{(n)}$, which is inverted to estimate the set of nth iteration interaction coefficient perturbations that best fits the observed spectra (weighted by inverse rms misfit). This regression is iterated to yield estimates of the interaction coefficients. Spectral fitting for self-coupling coefficients, hereafter referred to as self-coupled spectral fitting, employs just the $k=k'$ terms on the right of equation (1) in analyzing multiplet pairs. Generalized spectral fitting simultaneously estimates self-coupling and cross-coupling ($k\neq k'$) interaction coefficients.

The Impact of Coupling

It is reasonable to attempt to measure the normal mode signal of odd-degree structure only if such a signal is expected to be significant. Such expectations are well established if all of the following are true: that coupling provides sensitivity to odd-degree mantle structure comparable to typical modal sensitivity to even-degree structure; that inter-multiplet coupling considerably alters modal eigenvectors and eigenvalues; and that these perturbations resolveably impact data.

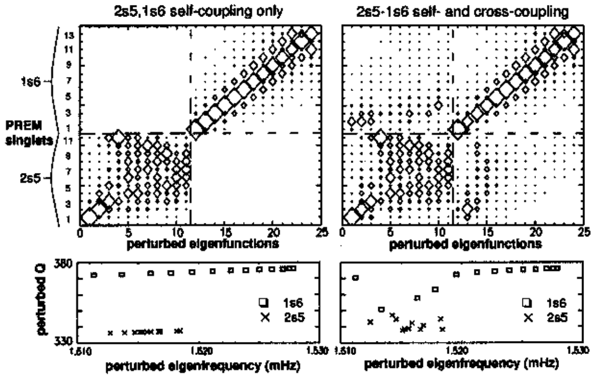

Figure 2. Top Row: Eigenvectors of ${}_2S_5-{}_1S_6$, produced by model S12_WM13, degrees 1-8. Polygon sizes in the eigenvector matrix indicate the relative contributions to each perturbed mode from each of the component PREM eigenfunction singlets. Bottom Row: Eigenvalues of ${}_2S_5-{}_1S_6$, separated into ${}_1S_6$ (squares) and ${}_2S_5$ (x's) groups according to the nature of the associated eigenvector. Center frequency and Q are generalized spectral fitting estimates: $\omega=1.5158$ and $Q=337$ for ${}_2S_5$; $\omega=1.5216$ and $Q=374$ for ${}_1S_6$.

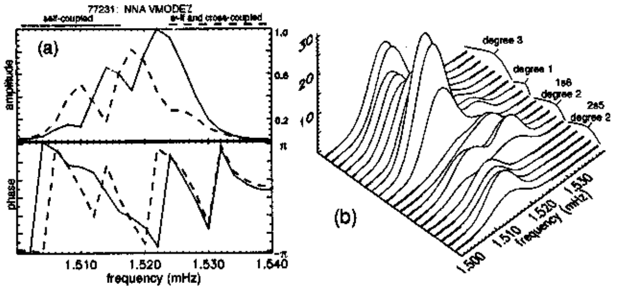

Figure 3. The impact of odd-degree structure on ${}_2S_5-{}_1S_6$ spectra. (a) Synthetic spectra of the multiplets produced by degrees 1-8 of S12_WM13. (b) Degree 1 and 2 1-norms of Frechet derivative amplitude spectra, $D^t_{s(kk')}(\omega_i)=\sum_j\mid\partial s_j^{(n)}(\omega_i)/\partial c^t_{s(kk')}\mid$, summed over 196 horizontal and vertical synthetic spectra of of ${}_2S_5{-}{}_1S_6$ for 10 events. Individual derivative spectra are moment-normalized before summing.

That the sensitivity of coupled overtones to mantle structure satisfies the first condition is demonstrated by Figure 1, which displays odd- and even-degree structure kernels $K_s(r)$ and $M_s(r)$ for the multiplet pair ${}_2S_5{-}{}_1S_6$. The odd- and even-degree kernels are of commensurate magnitudes through most of the mantle. Using the recent Harvard model S12_WM13 [Su *et al.*, 1993], Figure 2 demonstrates that the interaction coefficients resulting from the combination of odd-degree kernels and typical odd-degree Earth structure can meet the second condition above by strongly perturbing multiplet eigenvalues and eigenvectors.

Finally, that such perturbations should be resolvable in data is indicated both by large alterations in synthetic amplitude and phase spectra for indivual source-receiver pairs, exemplified by Figure 3a, and by the relative size of the average spectral impacts of perturbations to self- and cross-coupling interaction coefficients. These latter may be derived from the columns of the Frechet derivative matrix of equation (1). Figure 3b displays such spectra, and indicates that the impact of cross-coupling on ${}_2S_5-{}_1S_6$ is often stronger than that of self-coupling.

Spectral sensitivity to cross-coupling implies that odd-degree structure can bias estimates of structural constraints by self-coupled spectral fitting. This expectation is confirmed by a synthetic experiment. A suite

Table 1. Results of the Synthetic Experiment

mode	s	OUT vs IN		mode	s	OUT vs IN	
		corr	ratio			corr	ratio
${}_1S_5$	2	1.00	0.98	${}_2S_4$	2	1.00	1.04
${}_1S_5$	4	0.34	1.69	${}_2S_4$	4	0.45	1.35
${}_1S_6$	2	0.98	0.96	${}_2S_5$	2	0.93	0.79
${}_1S_6$	4	0.05	3.40	${}_2S_5$	4	0.54	3.09

Input model is S12_WM13, degrees 1-8. Self-coupled spectral fitting is used to estimate even degrees 2-8 interaction coefficients from a set of 195 horizontal and vertical synthetic seismograms of 17 events. The lateral correlations (corr) and ratios of rms amplitudes (ratio) of the OUTput and INput splitting functions at degrees 2 and 4 are shown.

Table 2. $\mathcal{X}^2$ Ratios of Data Regressions

structure degrees estimated	$\mathcal{X}^2$ ratio $_1S_5$-$_2S_4$	$_2S_5$-$_1S_6$
0 (PREM)	.630	.780
1	.380	.450
2	.240	.370
1,2	.190	.250
1,2,3	.150	.200
2,4	.190	.250
1,2,3,4	.120	.125
1,2,3,4,5	.115	.095

171 vertical recordings from 10 events were employed for $_1S_5$-$_2S_4$ regressions. 142 records were used for $_2S_5$-$_1S_6$.

of seismograms is synthesized from the self- and cross-coupled eigenvectors of the pairs $_1S_5-_2S_4$ and $_2S_5-_1S_6$, and self-coupled spectral fitting is applied to these synthetic data. Input and output coefficients are compared using lateral splitting functions, $F_{kk'}(\theta,\phi) = \sum_{s,t} c^t_{s(kk')} Y^t_s(\theta,\phi)$ [Giardini *et al.*, 1988]. The results of these comparisons, presented in Table 1, are summarized by two observations. First, the lateral correlation of estimated and input even-degree splitting functions is high only for degree 2. Second, the rms amplitudes of the estimated splitting functions at degree 4 are much greater than those for the input coefficients. Such biasing, which is even more pronounced at higher degrees, is the expected result of a regression which attributes the effects of both even- and odd-degree Earth structures to even-degree interaction coefficients alone.

Observations of Odd-degree Structure Coefficients

With PREM as an input model, both self-coupled and generalized spectral fitting for the interaction coefficients, degenerate frequencies, and Q's of the $_1S_5-_2S_4$ and $_2S_5-_1S_6$ pairs were performed on a set of approximately 150 vertical component spectra from 10 large events dating back to 1977. This set of high signal-to-noise recordings is dominated by data from the Northern Bolivian event of 1994.

In order to provide a qualitative assessment of the utility of generalized spectral fitting and the validity of the resulting estimates of even- and odd-degree constraints, three questions are posed. First, do the generalized spectral fitting regressions produce improved fits to data? If a measurable signal of odd-degree structure is being fit, misfits, as measured with $\mathcal{X}^2$ ratios [Ritzwoller *et al.*, 1988], should be smaller than those produced by self-coupled spectral fitting estimates for structure of comparable wavelengths. Second, are estimated odd-degree coefficients consistent existing constraints on aspherical structure, as defined, roughly, by the similarities and differences of recent Earth models? Third, does generalized spectral fitting yield improved agreement between estimated even-degree structure and model predictions, as forecast by the biasing effects observed in synthetics?

Table 3. Comparisons of Odd-degree Splitting Functions

mode	s	E vs H corr	E vs H ratio	E vs S corr	E vs S ratio	H vs S corr	H vs S ratio
$_1S_5$-$_2S_4$	1	0.96	1.25	0.53	1.68	0.63	1.33
$_1S_5$-$_2S_4$	3	0.92	1.10	0.92	1.05	0.96	0.95
$_2S_5$-$_1S_6$	1	0.96	1.44	0.68	1.76	0.49	1.22
$_2S_5$-$_1S_6$	3	0.78	1.19	0.75	1.15	0.95	0.96

Generalized spectral fitting estimates (E), Harvard (H) model S12_WM13 and Scripps (S) model SH.10c.17 are compared. 90% confidence levels for correlation at degrees 1 and 3 are .90 and .62, respectively.

Generalized spectral fitting produces significant improvements in best-fit $\mathcal{X}^2$ ratios. Table 2 lists misfits for suites of self-coupled and generalized spectral fitting regressions performed for $_1S_5-_2S_4$ and $_2S_5-_1S_6$ interaction coefficients. Estimates of degree 1 and 3 constraints reduce misfits more effectively than do estimates of even-degree constraints of degree 4 or greater.

The odd-degree interaction coefficients estimated using generalized spectral fitting appear consistent with the predictions of S12_WM13 and the Scripps model SH.10c.17 [Masters *et al.*, 1992], as illustrated by the splitting functions plotted in Figure 4, and by the statistics presented in Table 3. Lateral correlations to at least the Harvard model are above the 90% confidence level at degrees 1 and 3. Estimated splitting function amplitudes are somewhat greater than model predictions.

Finally, generalized spectral fitting yields improvements in the consistency of estimated even-degree structure coefficients with model predictions, as demonstrated by Table 4. The generalized spectral fitting estimates correlate markedly better with the models than do the self-coupling estimates, especially at degree 4, and rms amplitudes of estimated degree 4 splitting functions are reduced to values much nearer the model predictions.

These results appear to confirm the expectations created by the synthetic experiments, and lend validity to

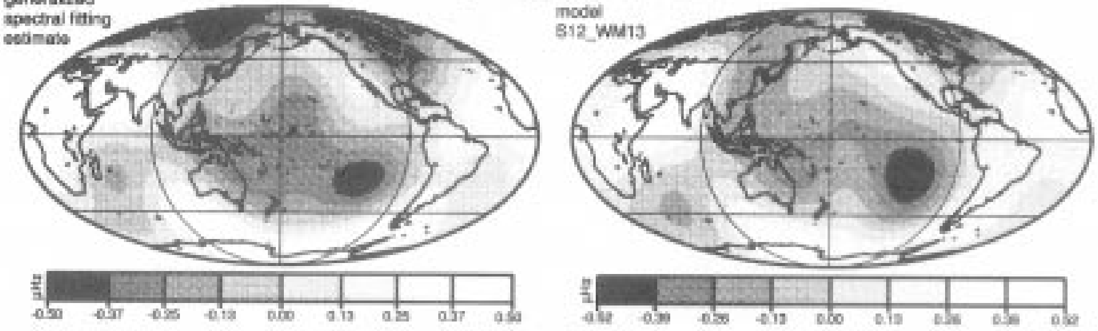

Figure 4. Estimated and predicted odd-degree splitting functions for $_1S_5-_2S_4$.

Table 4. Comparisons of Even-degree Splitting Functions

		self-coupled spectral fitting				generalized spectral fitting					
mode	s	E vs H		E vs S		E vs H		E vs S		H vs S	
		corr	ratio	corr	ratio	corr	ratio	corr	ratio	corr	ratio
${}_1S_5$	2	0.96	1.21	0.75	1.25	0.96	1.20	0.78	1.24	0.87	1.03
${}_1S_5$	4	0.41	2.62	0.51	1.97	0.73	1.59	0.57	1.20	0.25	0.75
${}_2S_4$	2	0.98	0.87	0.97	0.94	0.96	0.81	0.93	0.88	0.96	1.08
${}_2S_4$	4	0.33	1.49	0.60	1.11	0.63	1.11	0.52	0.83	0.51	0.74
${}_1S_6$	2	0.91	1.15	0.77	1.35	0.95	1.15	0.86	1.35	0.92	1.17
${}_1S_6$	4	0.26	3.40	0.21	2.50	0.59	2.31	0.69	1.70	0.29	0.74
${}_2S_5$	2	0.22	0.85	0.05	0.85	0.90	0.95	0.93	0.96	0.94	1.01
${}_2S_5$	4	0.19	6.67	-0.32	5.49	0.49	2.14	0.31	1.76	0.60	0.82

90% confidence levels for correlation at degrees 2 and 4 are .73 and .55, respectively.

Table 5. Odd-degree Interaction Coefficients

mode	c_1^0	$\mathrm{Re}(c_1^1)$	$\mathrm{Im}(c_1^1)$	c_3^0	$\mathrm{Re}(c_3^1)$	$\mathrm{Im}(c_3^1)$	$\mathrm{Re}(c_3^2)$	$\mathrm{Im}(c_3^2)$	$\mathrm{Re}(c_3^3)$	$\mathrm{Im}(c_3^3)$
${}_1S_5$-${}_2S_4$	1.30±.25	1.49±.15	-0.12±.15	-2.74±.30	1.03±.25	-0.15±.25	0.77±.20	-1.55±.25	-0.09±.25	-0.14±.20
${}_2S_5$-${}_1S_6$	0.71±.35	1.86±.30	-0.57±.30	-2.54±.60	-0.04±.45	-1.39±.55	0.75±.45	-2.24±.50	0.58±.45	0.09±.40

Uncertainties estimated with scaled standard deviations, similar to the method of Ritzwoller *et al.*, 1988.

the estimated odd-degree coefficients. Estimated degree 1 and 3 interaction coefficients for multiplet pairs ${}_1S_5-{}_2S_4$ and ${}_2S_5-{}_1S_6$ are presented in Table 5. These are the first constraints on odd-degree mantle structure to emerge from the analysis of long-period normal modes. Re-estimated even-degree constraints and higher order odd-degree coefficients will be presented as part of a more comprehensive study, in progress.

Conclusions

In addition to presenting the first estimates of interaction coefficients for odd-degree aspherical Earth structure, this study has demonstrated that:

- Cross-coupling of nearly degenerate multiplets by odd-degree structure can strongly perturb modal spectra.
- The effects of coupling can strongly bias purely self-coupling modal analyses.
- Generalization of the established spectral fitting technique to allow for simultaneous estimation of self-coupling and cross-coupling interaction coefficients permits both improved estimates of even-degree constraints on aspherical structure and estimation of odd-degree structural constraints.

Generalized spectral fitting has several immediate applications. The method will be employed in the analysis of approximately a dozen overtone pairs relevant to investigations of mantle structure. It will also be used in an attempt to improve estimates of the self- and cross-coupling interaction coefficients of strongly interacting spheroidal and toroidal fundamentals, and can be applied to along-branch suites of coupled multiplets.

Acknowledgments. We would like to thank the following individuals for providing the high quality digital seismic data employed in our regressions: Duncan Agnew at UCSD (IDA data), Peter Davis at UCSD (IRIS/IDA data), Genevieve Roult at IPG (GEOSCOPE data), Bob Woodward at the Albuquerque Seismic Lab (IRIS/GSN data), and Tim Ahern and colleagues at the IRIS Data Management Center at the University of Washington (GSN data). Data management software was provided by Danny Harvey and Dan Quinlan, and data preprocessing by Steven Smith, Anita Bowles, and Mark James. We thank Jeron Tromp and two additional reviewers for their constructive comments.

References

Dziewonski, A.M. and D.L. Anderson, Preliminary Reference Earth Model, Phys. Earth Planet. Inter., *25,* 297-356, 1981.

Giardini, D., X.-D. Li and J.H. Woodhouse, The splitting functions of long period normal modes of the Earth, J. Geophys. Res., *93,* 13716-13742, 1988.

Li, X.-D., D. Giardini and J. H. Woodhouse, Large-scale three-dimensional even-degree structure of the Earth from splitting of long-period normal modes, J. Geophys. Res., *91,* 551-577, 1991.

Masters, G., H. Bolton, and P. Shearer, Large-scale 3-dimensional structure of the mantle, Eos Trans. AGU, *73,* 201, 1992.

Ritzwoller, M.H., G. Masters and F. Gilbert, Constraining aspherical structure with low harmonic degree interaction coefficients: Application to uncoupled multiplets, J. Geophys. Res., *93,* 6369-6396, 1988.

Smith, M.F., and G. Masters, Aspherical Structure constraints from free oscillation frequency and attenuation measurements, J. Geophys. Res., *94,* 1953-1976, 1989.

Su, W., R.L. Woodward, and A.M. Dziewonski, Degree 12 Model of Shear Velocity Heterogeneity in the Mantle, J. Geophys. Res., *99,* 6945-6980, 1994.

Widmer, R., and G. Masters, Observably split multiplets – data analysis and interpretation in terms of large-scale aspherical structure, *Geophys. J. Int.*, *111,* 559-576, 1992.

Woodhouse, J.H., The coupling and attenuation of nearly resonant multiplets in the earth's free oscillation spectrum, Geophys. J. R. Astron. Soc., *61,* 261-283, 1980.

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(received February 22, 1995; accepted April 3, 1995.)