The feasibility of normal mode constraints on higher degree structures

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Abstract. Case studies of four multiplets (v_s-sensitive $_1S_7$ and $_1S_8$ and v_p-sensitive $_5S_5$ and $_5S_6$) are considered to test the accuracy of normal mode constraints on aspherical structure at degrees above 2 (4,6,8). Analyses of misfit, along-branch consistency, and consistency with existing mantle models argue for the feasibility of constraints on higher structural degrees. Preliminary interpretation indicates a relative insensitivity of data misfits to $dlnv_p/dlnv_s$ at degrees 6 and 8 in the lower mantle (perhaps due to inaccuracies in current δv_s models), and provides weak evidence of some decorrelation between δv_s and δv_p in the middle and lower mantle.

1. Introduction

Constraints on aspherical structure resulting from analyses of the Earth's normal modes have traditionally only been placed at the longest wavelengths, at spherical harmonic degrees 2 and 4 (e.g., Giardini et al., 1988; Ritzwoller et al., 1988; Li et al., 1991; Widmer and Masters, 1992). The growth in the Global Seismic Network and other global networks (e.g., GEOSCOPE, MEDNET) together with the occurrence of the large events of 1994 (especially the Northern Bolivian event on 6/9/94) have greatly improved the normal mode data set and hold promise for estimating structures previously unconstrained by normal modes. The purpose of this paper is to assess the current ability to estimate constraints on Earth structures at degrees above 2 by considering case studies of four well excited multiplets: the v_s-sensitive multiplets $_1S_7$ and $_1S_8$ and the v_p-sensitive multiplets $_5S_5$ and $_5S_6$. These case studies are a small part of a systematic study of mantle-sensitive multiplets to be reported elsewhere.

The method of normal mode analysis employed here is called 'spectral fitting' (e.g., Ritzwoller et al., 1988) and is described by Resovsky and Ritzwoller (1995) in this issue. Here, spectra for a given multiplet $k = (n, l)$ are iteratively fit with self-coupled regressions to estimate 'structure' or 'interaction coefficients', $_kc_s^t$, which are linearly related to aspherical structure at spherical harmonic degree s and azimuthal order t. As summaries of aspherical structure, structure coefficients provide tests of existing aspherical models (e.g., Ritzwoller and Lavely, 1995) and have been used as data

in structural inversions (e.g., Woodhouse and Dziewonski, 1989; Woodward and Masters, 1991; Masters et al., 1992; Ritzwoller and Wahr, 1994).

Quantitative assessment of the accuracy of structure coefficients is difficult since standard error analyses applied to normal mode regressions yield questionable uncertainty estimates (e.g., Ritzwoller et al., 1988). In an attempt to provide a qualitative assessment of structure coefficients above degree 2, we (1) compare the estimated coefficients with those predicted by two recent mantle models (SH.10c.17 of Masters et al., 1992 and S12_WM13 of Su et al., 1994), (2) discuss the consistency of the estimated coefficients for multiplets that sample the Earth similarly, and (3) investigate how the inclusion of higher degrees affects data misfits.

## 2. Estimation of Higher Degree Structure Coefficients

Although spectral fitting now is being applied systematically to many mantle-sensitive modes, we will discuss here application to only four multiplets: $_1S_7$, $_1S_8$, $_5S_5$ and $_5S_6$. Data sets used in each regression are taylored to the characteristics of each multiplet. The taylored data sets differ substantially for the $_1S$ and $_5S$ branches. The $_5S$ multiplets are vertically polarized and reside in the shadow of nearby fundamental modes, so that only vertical recordings from deep events (e.g., Fiji (3/9/94), Northern Bolivia) are used in order to inhibit fundamental mode amplitudes and, hence, interference with the $_5S$ regressions. By contrast, $_1S_7$ and $_1S_8$ are well excited on both vertical and horizontal components and are isolated from all interfering multiplets. This results in larger data sets for the $_1S$ modes, but we concentrate here on vertical recordings to optimize the signal-to-noise ratio. [203 recordings from 16 events (203, 16) for $_1S_7$, (221, 17) for $_1S_8$, (106, 5) for $_5S_5$, (120, 7) for $_5S_6$]. Each regression starts with all estimated coefficients at zero; that is, at PREM.

The existence of fundamental modes near to the $_5S$ modes as well as $_3S_7$ adjacent to $_5S_5$, complicate the $_5S$ regressions. This type of interference is the rule rather than the exception, and data and synthetic regressions indicate clearly that accurate estimation of higher degree coefficients requires that interfering modes be modeled in the regression. In the $_5S$ regressions, nearby spheroidal and toroidal fundamental synthetics coupled through the Coriolis force and aspherical structure from S12_WM13 are computed and removed from the spectrum ($_0S_{18}$ for $_5S_5$, $_0S_{21}$ and $_0T_{22}$ for $_5S_6$), but the degenerate frequency and Q of each of the fundamen-
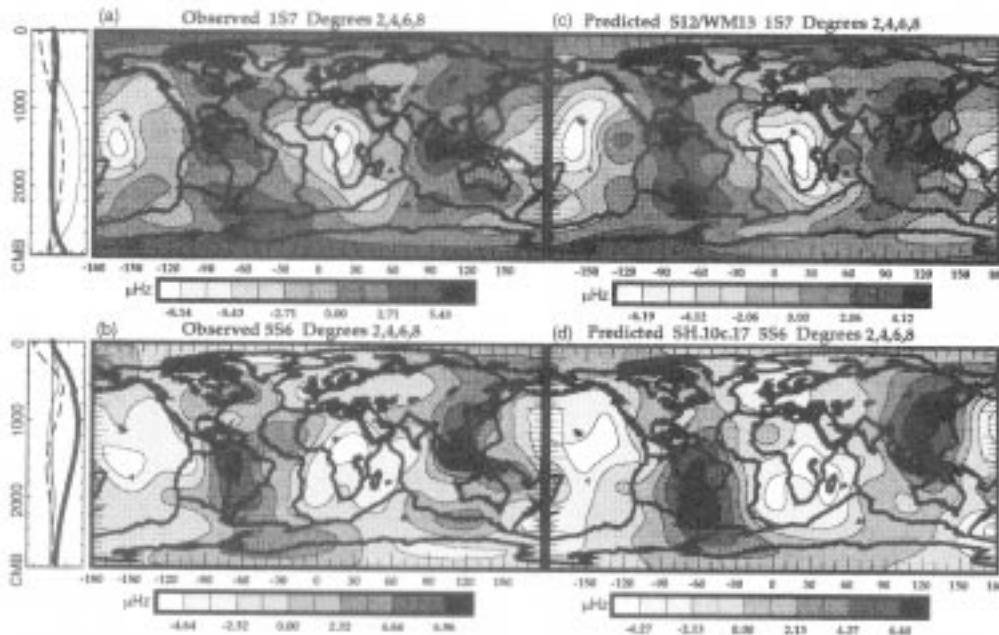
Figure 1. Splitting functions at degrees 2, 4, 6, and 8 for $_1S_7$ and $_5S_6$. Observed: (a) and (b). Predicted: (c) and (d). Units are μHz. Sensitivity kernels are shown at left as a function of depth in the mantle (thick solid line: v_p, thin solid line: v_s, dashed line: ρ).

tal modes are estimated. For $_5S_5$, the degenerate frequency, Q, and the degree 2 structure coefficients of $_3S_7$ are also estimated, but higher degree coefficients are fixed at those provided by S12_WM13. For each of the $_1S$ and $_5S$ multiplets, structure coefficients at even structural degrees $s = 2, 4, 6, 8$ are estimated. Estimated degenerate frequencies (mHz) and Q's are (1.65455, 437) for $_1S_7$, (1.79795, 443) for $_1S_8$, (2.70361, 579) for $_5S_5$, and (3.01136, 588) for $_5S_6$, with approximate uncertainties of ($\pm0.1\mu Hz$, ±25).

Estimated splitting functions for two example multiplets, $_1S_7$ and $_5S_6$, are shown in Figure 1. Space prohibits listing coefficient estimates. The plots of the integral kernels in Figure 1 illustrate the dominant sensitivity of $_1S_7$ to v_s in the lower mantle and of $_5S_6$ to v_p in the middle mantle. A predicted splitting function for each multiplet in Figure 1 is also shown, computed from the mantle model that fits the observations best: model S12_WM13 for $_1S_7$ and model SH.10c.17 for $_5S_6$.

3. Assessment of Higher Degree Structure Coefficients

Assessment of the accuracy of the estimated coefficients follows the three steps outlined in Section 1.

3.1 Comparison with Existing Models

Model predictions employ $dlnv_\rho/dlnv_s = 0.5$ in the lower mantle and $dlnv_p/dlnv_s = 0.8$ in the upper mantle. The lower mantle $v_p : v_s$ relationship is consistent with the arguments of Li *et al.* (1991) and the upper mantle value is consistent with the anharmonic zero-pressure value. $dln\rho/dlnv_s$ is taken from inversions that fit the geoid (Ritzwoller and Wahr, 1994): 0.5 in the upper mantle, 0.05 in the lower mantle. The choice of these values over conventional values of 0.25-0.40 throughout the mantle has little impact on the computed coefficients and no affect on the inferences drawn

here. No topography is placed on the transition zone boundaries nor on the CMB. Addition of topography on these boundaries from Ritzwoller and Wahr (1994) has no impact on our conclusions. Both S12_WM13 and SH.10c.17 possess topography on the crustal boundaries. Table 1 summarizes the correlations between the estimated and predicted coefficients, and between the coefficients predicted from the two models.
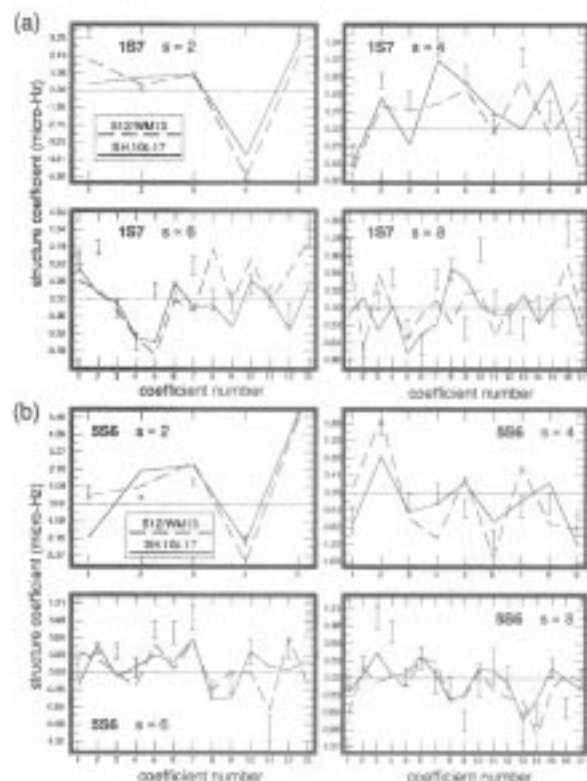
Figure 2. Comparison of observed and predicted structure coefficients at even structural degrees 2 - 8. Solid line: S12_WM13; dashed line: SH.10c.17. (a) $_1S_7$. (b) $_5S_6$. Units are μHz.

Table 1a. Consistency with Model Predictions ($_1S_7$)

s	correlation E:H	correlation E:S	correlation H:S	rms-ratio E/H	E/S
2	0.97 (99.8%)	0.89 (98.3%)	0.94 (99.4%)	1.18	1.39
4	0.85 (99.8%)	0.67 (97.4%)	0.37 (71.5%)	1.53	1.22
6	0.47 (91.1%)	0.48 (91.3%)	0.51 (94.2%)	0.92	1.29
8	0.59 (98.7%)	0.11 (31.4%)	0.08 (24.1%)	1.37	1.69

Estimates (E), Harvard (H) model S12_WM13, Scripps (S) model SH.10c.17. Confidence levels in parentheses.

Table 1b. Consistency with Model Predictions ($_5S_6$)

s	correlation E:H	correlation E:S	correlation H:S	rms-ratio E/H	E/S
2	0.97 (99.8%)	0.88 (97.8%)	0.90 (98.7%)	0.86	0.85
4	0.75 (98.8%)	0.83 (99.5%)	0.68 (97.0%)	0.74	1.15
6	0.66 (98.9%)	0.63 (98.2%)	0.35 (78.1%)	1.59	1.80
8	0.43 (91.3%)	0.78 (99.9%)	0.64 (99.4%)	1.32	1.69

See Table 1a caption.

Figures 2a and 2b display estimated structure coefficients and uncertainties with model predictions for $_1S_7$ and $_5S_6$. Uncertainties are estimated using the method described by Ritzwoller *et al.* (1988) in which relative coefficient uncertainties are estimated with a standard error analysis and are then scaled so that when they are added to the estimated coefficients a prescribed perturbation in the misfit is achieved. Coefficients numbering is such that $1, 2, 3, 4, 5, \ldots, 2s + 1$ should be interpreted as c_s^0, $\mathrm{Re}c_s^1$, $\mathrm{Im}c_s^1$, $\mathrm{Re}c_s^2$, $\mathrm{Im}c_s^2$, ..., $\mathrm{Re}c_s^s$, $\mathrm{Im}c_s^s$, where s denotes the degree of structure. These figures reveal a fairly good agreement between the observed and predicted coefficients, and also display the level of disagreement between predictions from the two mantle models. Coefficients are generally fit better in phase than in amplitude. Significant discrepancies between estimated and predicted coefficients are apparent, but are rare when predictions from the two models agree well. (Counterexamples are $\mathrm{Re}c_6^1$ and $\mathrm{Re}c_8^5$ for $_1S_7$.) Generally, a large discrepancy with one model is not apparent with the other model (e.g., $_1S_7$: $\mathrm{Im}c_4^1$, $\mathrm{Re}c_4^3$, $\mathrm{Im}c_6^6$, c_8^0, $\mathrm{Re}c_8^8$; $_5S_6$: c_2^0, $\mathrm{Re}c_4^1$, $\mathrm{Re}c_4^3$, $\mathrm{Im}c_6^5$, etc.).

Table 1 summarizes the fits between different sets of coefficients in terms of degree dependent correlations and rms-ratios. A rms-ratio is defined as the root-mean-square (rms) of a set of coefficients divided by the rms of another set of coefficients. S12_WM13 fits $_1S_7$ better than SH.10c.17, but SH.10c.17 fits $_5S_6$ better than S12_WM13. With a single exception ($s = 6$ for $_1S_7$), the observed coefficients correlate better with one or the other model than the two models correlate with one another. If the correlations between the coefficients predicted from these two models indicates uncertainties in the predicted coefficients, the observed:predicted correlations are probably as high as possible. The rms-ratios reveal that the estimated coefficients are somewhat enriched at shorter wavelengths relative to both models and that the coefficients predicted from S12_WM13 are generally larger than those from SH.10c.17.

3.2 Along-Branch Consistency

Within certain ranges the sensitivity kernels of these multiplets to aspherical structure change smoothly along dispersion branches. Thus, the structure coefficients themselves are likely to change smoothly along each branch. Confidence levels of correlations between $_1S_7$ and $_1S_8$ are >99.9% ($s = 2, 4$), 77.9% ($s = 6$), 99.2% ($s = 8$) and between $_5S_5$ and $_5S_6$ are >99.9% ($s = 2$), 99.6% ($s = 4$), 90.0% ($s = 6$), 83.8% ($s = 8$). The relatively low correlations of $_5S_5$ with $_5S_6$ apparent at degrees 6 and 8 may indicate errors in the $_5S_5$ coefficients due to inaccurately modeling the interference from $_3S_7$. This test is weakened by the fact that low

correlations do not necessarily indicate errors in the coefficients. For example, the correlation of the $_5S_5$ and $_5S_6$ coefficients predicted from the model SH.10c.17 at $s = 8$ is only 0.43. This results when the values of the coefficients represent a balance between competing contributions in the upper and lower mantles. The integral kernels of $_5S_5$ and $_5S_6$ are sufficiently different to affect this balance. Therefore, high along-branch consistency lends support to the accuracy of the estimates, but low along-branch consistency, in itself, does not imply that the coefficient estimates are inaccurate.

3.3 Analysis of Misfit

Figures 3a and 3b display average misfits for $_5S_6$ and $_1S_7$ for both the estimated and predicted coefficients (from the model that best fits each multiplet). Average misfits for the estimated coefficients are quite low. Amplitude errors less than 5% and phase errors less than $5°$ are common for these well excited multiplets.

We represent misfit quantitatively for a given recording with rms-residual-ratios. The residual is the difference between the observed spectrum and the spectrum predicted from a set of structure coefficients. The rms-
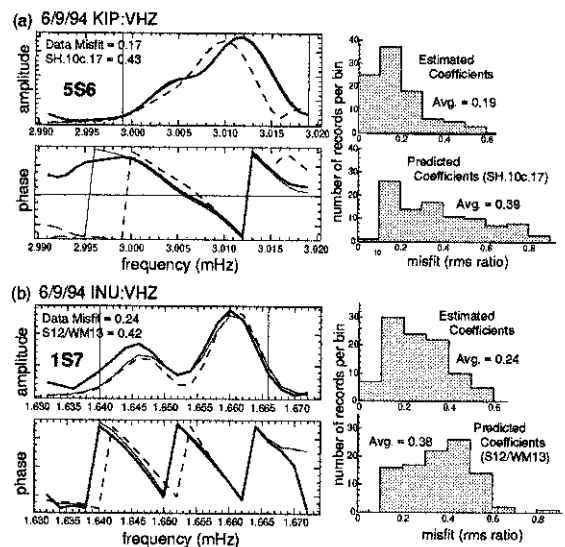
Figure 3. Examples of data misfit for $_5S_6$ and $_1S_7$. (a) $_5S_6$. At left: data amplitude and phase (bold solid lines) and synthetic amplitude and phase for the estimated degree 2 - 8 coefficients (thin solid lines) and those computed from SH.10c.17 (dashed lines). Observed and predicted amplitudes are indistinguishable at this scale. At right: histograms summarizing the rms-residual-ratios for the 100 highest signal-to-noise recordings, represented as the number of recordings in each 0.1 misfit increment bin. Average rms-residual-ratios are displayed. (b) The same as (a) but for $_1S_7$ and the use of S12_WM13.

residual-ratio is the ratio of the rms of the residual divided by the rms of the data in the fitting window. To represent average misfit, we have chosen the 100 highest signal-to-noise recordings for each multiplet. The average misfit defined in this way for $_5S_6$ is 0.19 and for $_1S_7$ is 0.24. Misfits for $_5S_5$ and $_1S_8$ are similar to $_5S_6$ and $_1S_7$, respectively. Since the $_5S$ modes are better excited by the Northern Bolivian event than the $_1S$ modes, the average misfit is worse for the $_1S$ modes. The inclusion of higher degree coefficients plays a substantial role in the data fit. The degree dependence of the RMS-residual-ratios for the estimated coefficients is as follows: $_5S_6$: 0.85 (s=0, RH-model), 0.41 (s=2), 0.33 (s=4), 0.28 (s=6), 0.19 (s=8); $_1S_7$: 0.81 (s=0, RH-model), 0.40 (s=2), 0.33 (s=4), 0.31 (s=6), 0.24 (s=8).

Histograms displaying the distribution of misfits for the 100 high signal-to-noise recordings in terms of the number of records per 0.1 misfit bin are also shown in Figures 3a and 3b. For example, 25 of the 100 recordings for $_5S_6$ have an rms-residual-ratio between 0.0 and 0.1, 35 are between 0.1 and 0.2, and so forth. Misfits from model predictions are considerably worse than from the estimated coefficients. (0.39 for $_5S_6$ with SH.10c.17 and 0.38 for $_1S_7$ with SH_WM13.) Although, as described above, the agreement between the observed and predicted coefficients is quite good, the differences affect misfit appreciably and should be taken seriously.

4. Discussion and Conclusions

From the evidence presented in Section 3, there are strong reasons to be optimistic about the estimation of normal mode structure coefficients at degrees 4, 6, 8 for isolated multiplets. These conclusions are borne out by analyzing a number of multiplets in addition to those considered here. The high correlations with existing models summarized in Table 1 are particularly compelling, but improvement in misfit afforded by the estimated coefficients over predicted coefficients indicates that discrepancies should be taken seriously. On a cautionary note, experience with both data and synthetic regressions clearly indicates that the estimation of higher degrees requires interfering modes to be modeled in the regression.

Because the estimated coefficients fit the data significantly better than model predictions, it is likely that they comprise new information about δv_s and δv_p structures in the middle and lower mantle. Preliminary experiments to estimate $dlnv_p/dlnv_s$ in the lower mantle imply that the $_5S$ degree 6 and 8 coefficients are consistent with the reduced ratio of 0.5 espoused by Li et al. (1991), but are also consistent with the anharmonic zero-pressure value of 0.8. This insensitivity may result from current inaccuracies in the man-

tle models at degrees 6 and 8. That current δv_s models fit the v_s-sensitive $_1S$ modes somewhat better than the v_p-sensitive $_5S$ modes may be evidence for a weak decorrelation of δv_s and δv_p in the lower mantle, but may simply arise from the greater sensitivity of the $_5S$ modes to smaller mid-mantle structures.

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